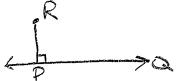
Proportions in the Right Triangle

Whenever the sun is shining, any object casts a shadow. If the sun were directly overhead, the projection of the object would be suggested by its shadow.

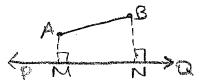
Def: projection of a point on a line- the foot of the perpendicular drawn from that point to the line

ex: the projection of point R on line PQ is P



<u>Def</u>: <u>projection of a segment on a line</u>- (when the segment is not perpendicular to the line) the segment whose endpoints are the projections of the endpoints of the given line segment on the line

ex: the projection of segment AB on line PQ is segment MN



<u>Thm</u>: The altitude to the hypotenuse of a right triangle divides the triangle into two smaller triangles that are each similar to each other and to the original triangle as well.

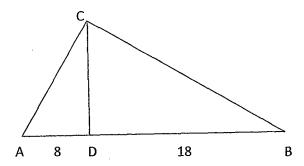
Proof:

Prove: AABC~LACO~LCBD

1. * ACB is a rt. * 1. given 2.

	If a right triangle is the mean proportional between the length of the nuse and the length of the hypotenuse.
<u>Proof:</u>	
2	
	le to the hypotenuse of a right triangle is the mean proportional of the legs on the hypotenuse.
<u>l'roof</u> :	

<u>Ex1</u>:



Find:
a) AC
b) BC
c) CD

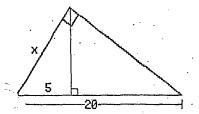
 $\underline{\mathtt{Ex2}}$: The altitude to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of one segment is 5 inches more than the other. If the length of the altitude is 6 inches, find the length of the hypotenuse.

Right Triangle Proportions

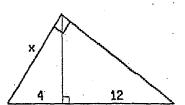
Notes/diagrams:

Examples:

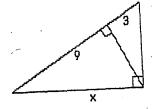
Solve for x:



Solve for x:

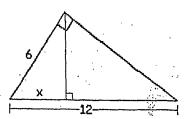


Solve for X:



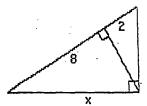
2

Solve for x:



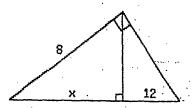
(4)

Solve for x:

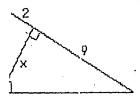


6

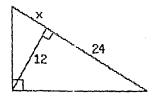
Solve for x:



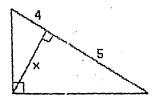
lve for x:



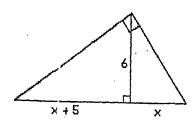
Bolve for x:

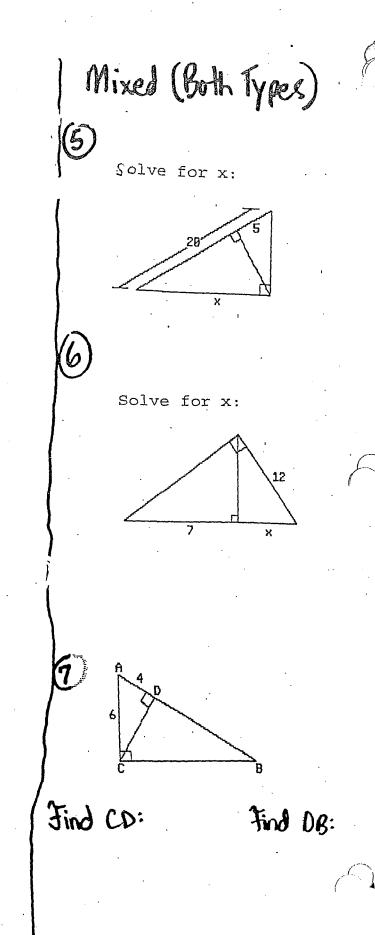


Solve for x:



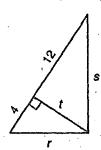
Solve for x:



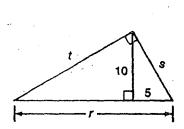


In Exercises 1 to 3, find the values of r, s, and t.

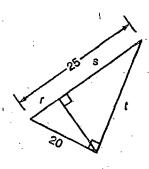
1.



2.



3.



In Exercises 4 to 9, in right triangle JKL, angle JKL is the right angle and $\overline{KH} \perp \overline{JL}$.

4. If JH = 4 and HL = 16, find KH.

5. If JH = 5 and HL = 4, find KL.

6. If JH = 8, JL = 20, find KH.

7. If KL = 18, JL = 27, find JK.

8. If JK = 14, HL = 21, find JH.

9. If KH = 12, JL = 40, find JK (assume \overline{JK} is the shorter leg of right $\triangle JKL$).

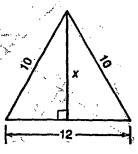
10. The altitude drawn to the hypotenuse of a right triangle divides the hypotenuse into segments such that their lengths are in the ratio of 1:4. If the length of the altitude is 8, find the length of:

(a) Each segment of the hypotenuse.

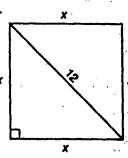
(b) The longer leg of the triangle.

In Exercises 11 to 15, find the value of x.

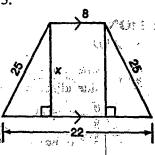
11.



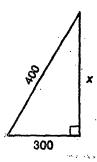
12.



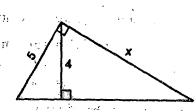
13



14.



15.



	; ;	
	t E	
	#	
	\$ 5	

Similarity in Right Triangles; The Pythagorean Theorem

For use after Section 8-2

Simplify.

2.
$$2\sqrt{50}$$

3.
$$\sqrt{20} \cdot \sqrt{6}$$

4.
$$\frac{2}{\sqrt{5}}$$

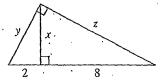
5.
$$\sqrt{\frac{1}{3}}$$

6.
$$\left(\frac{\sqrt{3}}{3}\right)^2$$

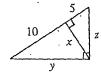
Find the geometric mean between the two numbers.

Each diagram shows a right triangle with the altitude drawn to the hypotenuse. Find the values of x, y, and z.

10.



11.



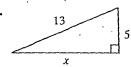
$$\dot{x} =$$
______, $\dot{y} =$ _______, $\dot{z} =$ _______

Find the value of x.

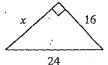
12.

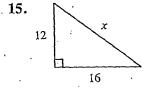


13.



14

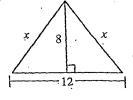




16.



17.



$$x =$$

- 18. A rectangle has length 2.4 m and width 0.7 m. Find the length of a diagonal.
- 19. A square has perimeter 12 cm. Find the length of a diagonal.
- 20. The diagonals of a rhombus have lengths 12 and 16. Find the perimeter of the rhombus.

Converse of Pythag Special Right Triang

rean Theorem; es

For use after Section 8-4

Tell whether a triangle with sides c or obtuse. If a triangle can't be for

1. 5, 6, 7

4. 8, 15, 17

7. 6, 8, 10

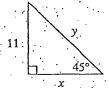
the given lengths is acute, right, ied, write not possible.

3. 2.0, 2.1, 2.7 _

6. 9, 12, 15 ___

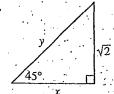
9. 9, 40, 41

Find the missing lengths.



$$y =$$

11.

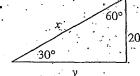


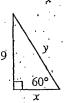
12.





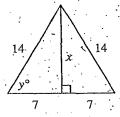
13.





15.

16.



17.

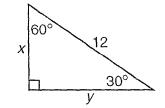
- 18. An equilateral triangle has si is of length 16. Find the length of an altitude.
- 19. Find the perimeter of a sque with diagonal of length 12.

20. An equilateral triangle has a altitude of length $5\sqrt{3}$. Find the perimeter. _

.Name:
30-60-90 Special Right Triangle
Questions 1 through 10 refer to the following:
Use the information marked on the figure to find the value of x and y .
16/
1)
<u>30°</u> <u>y</u>



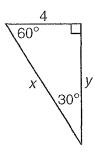
3)



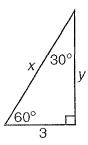
4)

y 30° × 60

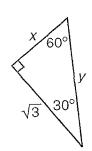
5)



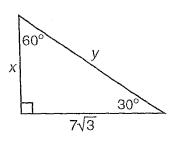
6)



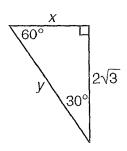
7)



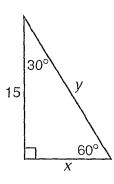
8)



9)



10)



Can these be the sides of a right triangle?

- 11) Which set of numbers represents the lengths of the sides of a right triangle?
 - A) {4,36,40}
- B) {2,18,20}
- C) $\{4,6,\sqrt{40}\}$
- D) $\{2,6,\sqrt{40}\}$

- 12) Which set of numbers represents the lengths of the sides of a right triangle?
 - A) {5,5,10}

- B) {5,12,13}
- C) {2,3,6}

- D) {4,5,6}
- 13) Which set of numbers could not represent the lengths of the sides of a right triangle?
 - A) {5,12,13}
- B) {8,15,17}
- C) {3,4,5}

D) {6,9,12}

- (14) Which set of numbers could represent the lengths of the sides of a right triangle?
 - A) {8,15,17}
- B) {7,9,11}

C) {7,8,12}

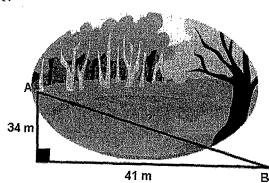
D) {5,7,8}

15)	10, 24 and 26 are the lengths of the sides of	triangle. Determine whether the triangle is a right triangle.
16)	5, 6 and 7 are the lengths of the sides of a t	ngle. Determine whether the triangle is a right triangle.
17)	7, 8 and 9 are the lengths of the sides of a 1	${\sf Ing}[e]$ Determine whether the triangle is a right triangle.
18)	6, 8 and 10 are the lengths of the sides of a	angle. Determine whether the triangle is a right triangle.
19)	7, 24 and 25 are the lengths of the sides of	Iriangle. Determine whether the triangle is a right triangle.
20)	3, 4 and 5 are the lengths of the sides of a	ungle. Determine whether the triangle is a right triangle.
21)	2, 18 and 20 are the lengths of the sides of	triangle. Determine whether the triangle is a right triangle.

λ7	
Name:	

Pythagorean Theorem

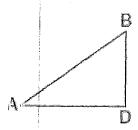
1.



To get from point A to point B you must avoid walking through a pond. To avoid the pond, you must walk 34 meters south and 41 meters east. To the nearest meter, how many meters would be saved if it were possible to walk through the pond?

- 2. The lengths of the sides of a right triangle can be
 - 1) 9,12,15
 - 2) 8,10,13
 - 3) 5,5,10
 - 4) 4,5,6

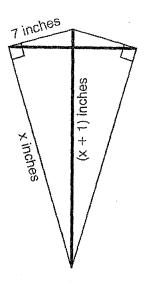
3. In the diagram below of $\triangle ADB$, $m\angle BI = 90$, $AD = 5\sqrt{2}$, and $AB = 2\sqrt{15}$.



What is the length of \overline{BD} ?

- 4. Which set of numbers could *not* repre nt the lengths of the sides of a right triangle?
 - 1) $\{1,3,\sqrt{10}\}$
 - 2) {2,3,4}
 - 3) {3,4,5}
 - 4) {8,15,17}

5. As shown in the diagram below, a kite needs a vertical and a horizontal support bar attached at opposite corners. The upper edges of the kite are 7 inches, the side edges are x inches, and the vertical support bar is (x + 1) inches.



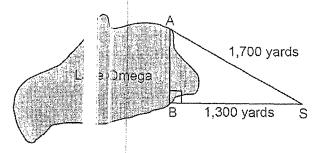
What is the measure, in inches, of the vertical support bar?

6. The legs of an isosceles right triangle each measure 10 inches. What is the length of the hypotenuse of this triangle, to the *nearest tenth of an inch*?

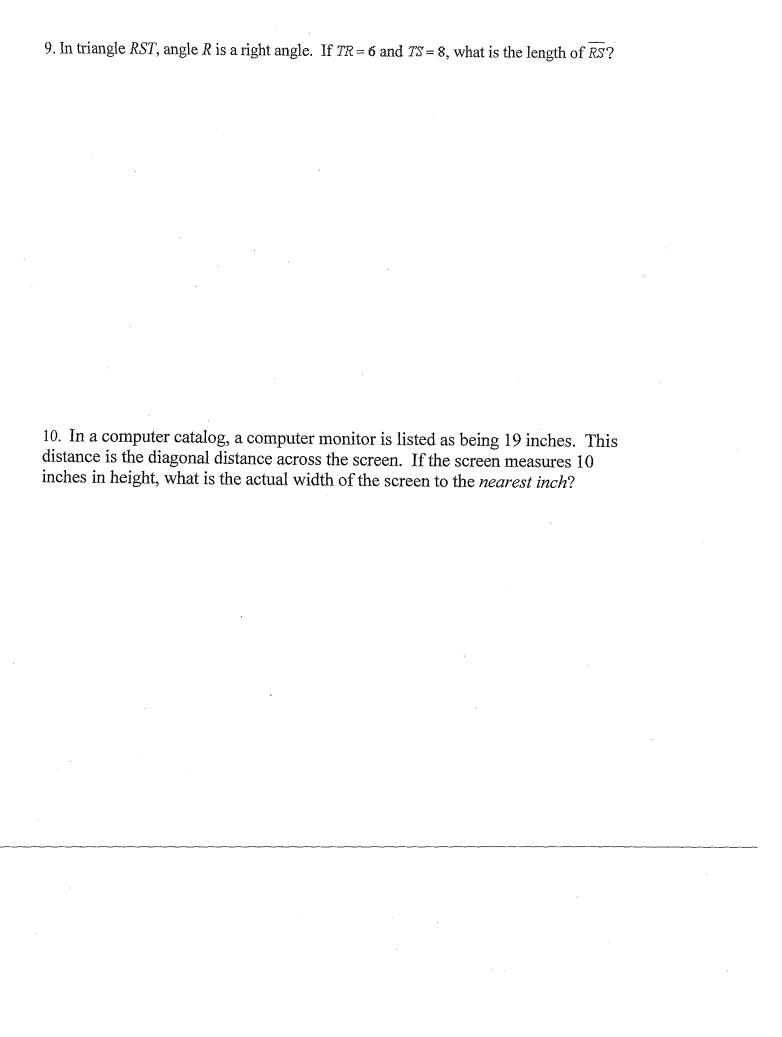
7. The length of one side of a square is 13 feet. What is the length, to the nearest foot, of a diagonal of the square?

8. Campsite A and campsite B are locate—directly opposite each other on the shores of Lake campsite A is 1,700 yards from his position.

Omega, as shown in the diagram low. The two campsites form a right triangle with Sem's position, S. The distance f in campsite B to Sam's position is 1,300 yards, and

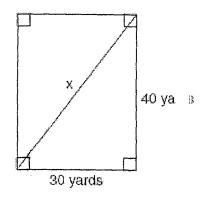


What is the distance from campsi A to campsite B, to the nearest yard?



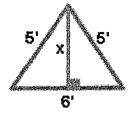
11. A suitcase measures 24 inc so long and 18 inches high. What is the diagonal length of the suitcase the nearest tenth of a foot?

12.



Tanya runs diagonally across a rectangular field that has a length of 40 yards and a width of 30 yards, as shown in the diagram.

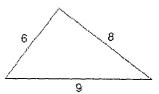
What is the length of the diagonal, in yards, that Tanya runs?



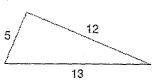
14.

Do the following lengths form a right triangle?

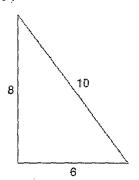
1)



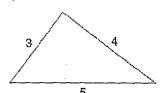
2)



3)

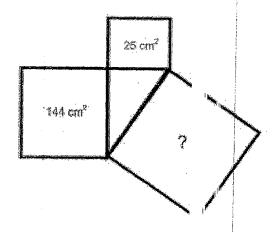


4)



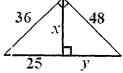
15. The accompanying picture sho s 3 squares and 1 triangle.

Find the area of the missing squ e. SHOW YOUR WORK!!!



16.

Compare the quantity in Column 4 with the quantity in Column B.



Column A

Column B

:¢

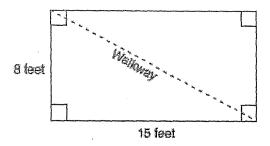
- [A] The quantity in Column A i greater.
- [B] The quantity in Column B is greater.

[C] The two quantities are equa

 ν

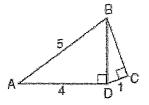
[D] The relationship cannot be termined on the basis of the information supplied.

17. Nancy's rectangular garden is represented in the diagram below.

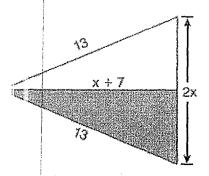


If a diagonal walkway crosses her garden, what is its length, in feet?

18. In the accompanying diagram of right triangles ABD and DBC, AB = 5, AD = 4, and CD = 1. Find the length of \overline{BC} , to the nearest tenth.

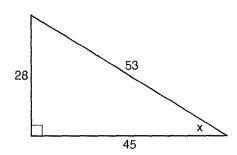


- 19. The diagram below shows a pennant measure 13, the altitude is x + 7, ar
 - the shape of an isosceles triangle. The equal sides each the base is 2x.

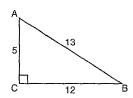


What is the length of the base?

1) Which ratio represents $\sin x$ in the right triangle shown below?

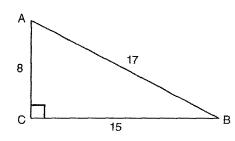


- 1) $\frac{28}{53}$
- 2) $\frac{28}{45}$
- 3) $\frac{45}{53}$
- 4) $\frac{53}{28}$
- 3) The diagram below shows right triangle ABC.



Which ratio represents the tangent of $\angle ABC$?

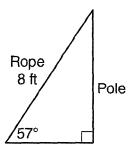
- 1) $\frac{5}{13}$
- 2) $\frac{5}{12}$
- 3) $\frac{12}{13}$
- 4) 12
- 5) Right triangle ABC has legs of 8 and 15 and a hypotenuse of 17, as shown in the diagram below.



The value of the tangent of $\angle B$ is

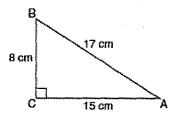
- 1) 0.4706
- 3) 0.8824
- 2) 0.5333
- 4) 1.8750

2) An 8-foot rope is tied from the top of a pole to a stake in the ground, as shown in the diagram below.



If the rope forms a 57° angle with the ground, what is the height of the pole, to the *nearest tenth of a foot*?

- 1) 4.4
- 2) 6.7
- 3) 9.5
- 4) 12.3
- 4) Which equation shows a correct trigonometric ratio for angle A in the right triangle below?



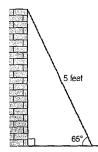
- $\sin A = \frac{15}{17}$
- $\tan A = \frac{8}{17}$
- $\cos A = \frac{15}{17}$
- $\tan A = \frac{5}{8}$
- 6) In $\triangle ABC$, the measure of $\angle B = 90^{\circ}$, AC = 50, AB = 48, and BC = 14. Which ratio represents the tangent of $\angle A$?
- 1) $\frac{14}{50}$

3) $\frac{48}{50}$

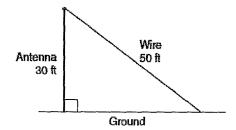
2) 14

4) $\frac{48}{14}$

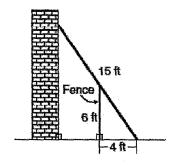
7) As shown in the diagram below, a ladder 5 feet long leans against a wall and makes an angle of 65° with the ground. Find, to the nearest tenth of a foot, the distance from the wall to the base of the ladder.



3) A communications company is building a 30-foot antenna to carry cell phone transmissions. As shown in the diagram below, a 50-foot wire from the top of the antenna to the ground is used to stabilize the antenna.



- Find, to the nearest degree, the measure of the angle that the wire makes with the ground.
- 9) In the accompanying diagram, the base of a 15-foot ladder rests on the ground 4 feet from a 6-foot fence.
 - a) If the ladder touches the top of the fence and the side of a building, what angle, to the *nearest degree*, does the ladder make with the ground?

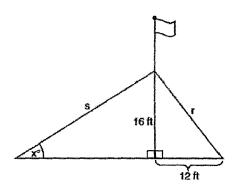


b) Using the angle found in part a, determine how far the top of the ladder reaches up the side of the building, to the nearest foot.

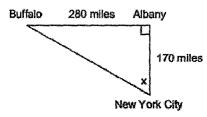
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:	
:	

-

10) The accompanying diagram shows a flagpole that stands on level ground. Two cables, r and s, are attached to the pole at a point 16 feet above the ground. The combined length of the two cables is 50 feet. If cable r is attached to the ground 12 feet from the base of the pole, what is the measure of the angle, x, to the nearest degree, that cable s makes with the ground?

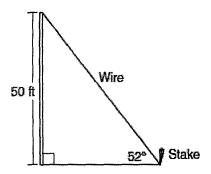


- 11) As seen in the accompanying diagram, a person can travel from New York City to Buffalo by going north 170 miles to Albany and then west 280 miles to Buffalo.
 - a) If an engineer wants to design a highway to connect New York City directly to Buffalo, at what angle, x, would she need to build the highway? Find the angle to the nearest degree.



b) To the nearest mile, how many miles would be saved by traveling directly from New York City to Buffalo rather than by traveling first to Albany and then to Buffalo?

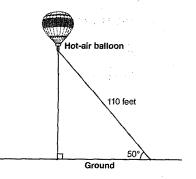
12) A stake is to be driven into the ground away from the base of a 50-foot pole, as shown in the diagram below. A wire from the stake on the ground to the top of the pole is to be installed at an angle of elevation of 52° .



How far away from the base of the pole should the stake be driven in, to the *nearest foot*? What will be the length of the wire from the stake to the top of the pole, to the *nearest foot*?

13) A hot-air balloon is tied to the ground with two taut (straight) ropes, as shown in the diagram below. One rope is directly under the balloon and makes a right angle with the ground. The other rope forms an angle of 50° with the ground.

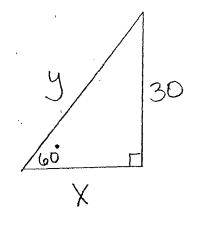
Determine the height, to the *nearest foot*, of the balloon directly above the ground. Determine the distance, to the *nearest foot*, on the ground between the two ropes.



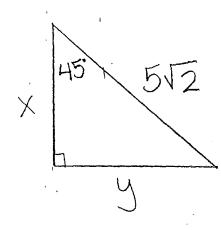
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Special Right Triangles

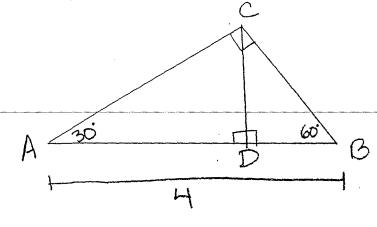
Find X+y:



2.

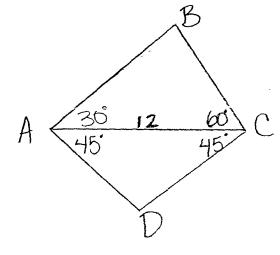


3.



Find:

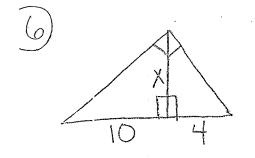
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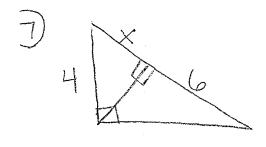


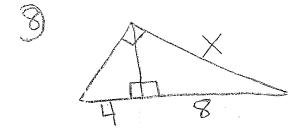
Find:



6) 9,10,15



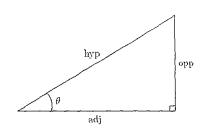




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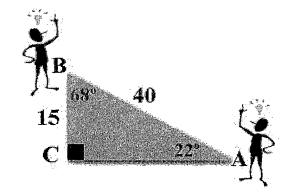
•

.



$$\sin \theta = \frac{opposite}{hypotenuse} \cos \theta = \frac{adjacent}{hypotenuse} \tan \theta = \frac{opposite}{adjacent}$$

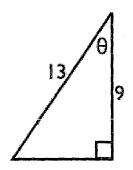
1.



- a) Sin A =
- a) Sin B =
- b) $\cos A =$
- b) $\cos B =$
- c) Tan A =
- c) Tan B =

Do you notice anything about the above answers?

2. If $\cos \theta = \frac{9}{13}$, what is the Sin of the missing angle?



3. In right triangle ABC with the right angle at C, $\sin A = 2x + 0.1$ and $\cos B = 4x - 0.7$. Determine and state the value of x. Explain your answer.

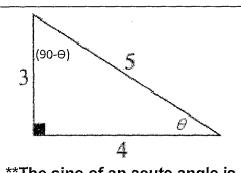
4. In right triangle DOG with the ght angle at O, $\sin D = 3x - .7$ and $\cos G = 8x - 23$. Determine and state the value of \Rightarrow Explain your answer.

5. In right triangle HAT with the sight angle at A, $\sin \mathbf{H} = x + .2$ and $\cos T = 7x - .1$. Determine and state the value of x. Explain our answer.

6. In right triangle TOE with the ght angle at O, $\sin T = 7x - .6$ and $\cos E = 4x + 9$. Determine and state the value of Explain your answer.

7. In right triangle SAM with the right angle at A, $\sin S = 5x - .7$ and $\cos M = 9x - .46$. Determine and state the value of \therefore Explain your answer.

CoFunctions



**The sine of an acute angle is equal to the cosine of its complement.

**The cosine of an acute angle is equal to the sine of its complement.

In the diagram at the left, the measures of the angles designated by Θ and $(90 - \theta)$ add to 90° .

These angles are complementary angles.

$$\sin \theta = \frac{3}{5}$$
 and

$$\sin \theta = \frac{3}{5}$$
 and $\cos(90 - \theta) = \frac{3}{5}$

$$\cos \theta = \frac{4}{5}$$
 and $\sin(90 - \theta) = \frac{4}{5}$

$$\sin(90-\Theta) = \frac{4}{5}$$

**The value of a trig function of an angle equals the value of the cofunction of the omplement of the angle.

Cofunction Identities, in degrees:

Sine and cosine are cofunctions.

$$\sin \theta = \cos(90^{\circ} - \theta)$$

$$\cos\theta = \sin(90^\circ - \theta)$$

Notice the connection of the letters C & O

- * sine and cosine cofunctions
- * complementary



Examples:

- 1. If $\sin 6A = \cos 9A$, then $m \angle A$ is equal to
 - a) 6
- c) 54
- b) 36
- d) $1\frac{1}{2}$
- 2. If $sin(A-30)^\circ = cos 60^\circ$, the number c degrees in the measure of angle A is
 - a) 30
- c) 90
- b) 60
- d) 120
- 3. If $cos(x + 30^\circ) = sin x$, a measure of ngle x is
 - a) 15º
- c) 45º
- b) 30º
- d) 60º
- 4. If $\sin 2A = \cos 3A$, then $m \angle A$ is
 - a) $1\frac{1}{2}$
- c) 18
- b) 5
- d) 36
- 5. If $cos(2x-1)^{\circ} = sin(3x+6)^{\circ}$, then the value of x is
 - a) -7
- c) 35
- b) 17
- d) 71

Geometry Pre-IB Period _____

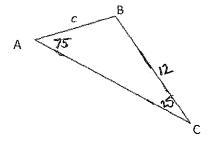
Law of Sines

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

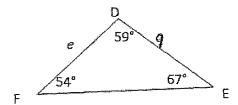
$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Practice:

1. For \triangle ABC find c to the nearest hundredth.

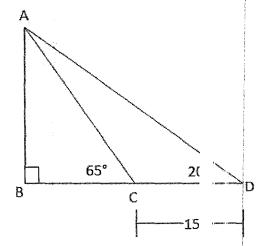


2. For $\triangle DEF$ find e to the nearest hundredth.

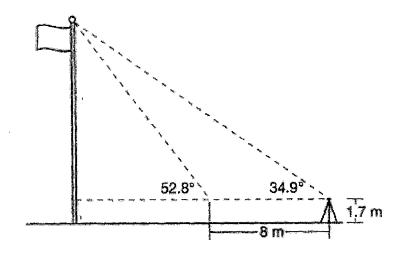


3. For $\triangle DEF$, e = 34, $m \angle D = 3^{-p}$, $m \angle E = 72^{\circ}$, and $m \angle F = 72^{\circ}$ Find e to the nearest whole degree.

4. For the figure below find BC to energest whole number. CD=15.

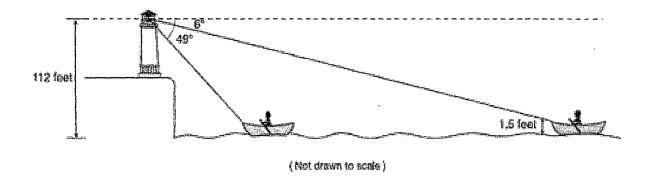


Cathy wants to determine the height of the flagpole shown in the diagram below. She uses a survey instrument to measure the angle of elevation to the top of the flagpole, and determines it to be 34.9°. She walks 8 meters closer and determines the new measure of the angle of elevation to be 52.8°. At each measurement, the survey instrument is 1.7 meters above the ground.



Determine and state, to the nearest tenth of a meter, the height of the flagpole.

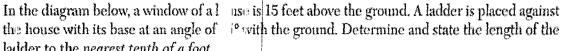
As shown below, a cance is approaching a lighthouse on the coastline of a lake. The front of the cance is 1.5 feet above the water and an observer in the lighthouse is 112 feet above the water.

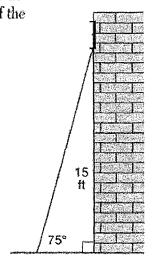


At 5:00, the observer in the lighthouse measured the angle of depression to the front of the canoe to be 6°. Five minutes later, the observer measured and saw the angle of depression to the front of the canoe had increased by 49°. Determine and state, to the nearest foot per minute, the average speed at which the canoe traveled toward the lighthouse.

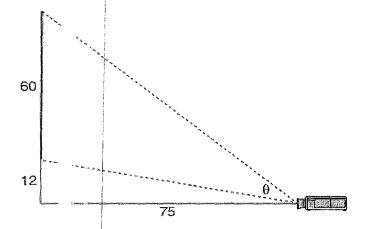
4. ladder to the nearest tenth of a foot.

5.





As modeled below, a movi is projected onto a large outdoor screen. The bottom of the 60-foot-tall screen is 12 fee off the ground. The projector sits on the ground at a horizontal distance of 75 feet from the



Determine and state, to the nearest tenth of a degree, the measure of θ , the projection angle.

Right Triangle Trigonometry Formula Sheet

I. Pythagorean Theorem: Used to find a side when 2 sides are given

$$a^2 + b^2 = c^2$$

*c must be the hypotenuse

II. In a right triangle, to find a side or an angle when a side **and** an angle are given use:

$$\sin \theta = \frac{Opposite}{Hypotenuse}$$

$$\cos \theta = \frac{Adjacent}{Hypotenuse}$$

Tan
$$\theta = \frac{Opposite}{Adjacent}$$

III. Special Right Triangle Rations:

$$45^{\circ} - 45^{\circ} - 90^{\circ}$$
 Ratio X : X : $X\sqrt{2}$

$$30^{\circ} - 60^{\circ} - 90^{\circ}$$
 Ratio X : $X\sqrt{3}$: 2X

IV. **Rule:** In a right triangle the Sin of one acute angle is equal to the cosine of the other acute angle.

Ex. In \triangle ABC, if <C is the right angle then Sin A = Cos B

7. **CoFunctions:** Sine and cosine are cofunctions. Think complementary.

$$\sin \theta = \cos(90^{\circ} - \theta)$$

$$\cos\theta = \sin(90^{\circ} - \theta)$$

Non Right Triangle Formulas

*Remember: little letters are sides; capital letters are angles

I. Law of Sines =

Used when given 2 sides and 1 angle trying to get angle opposite given side OR

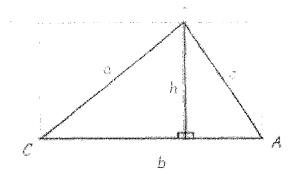
given 2 angles and 1 side trying to get side opposite given angle

II. Area of a Non Right Triangle = Area = $\frac{1}{2} \cdot c \cdot b \cdot \sin(A)$

*Think $\frac{1}{2}$ side 1 side 2 sin of the included angle

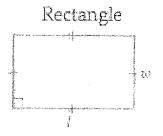
Area of a Non-Right Triangle:

Derivation:



$$A = \frac{1}{2}ab\sin C \qquad ("SAS")$$

Area formulas for common shapes



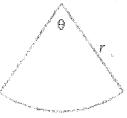
$$A = \hbar \omega$$

Circle



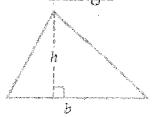
$$\Lambda = \pi r^2$$

Sector



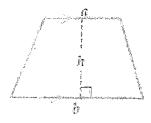
$$A = \frac{\theta}{360} \times \pi r^2$$

Triangle



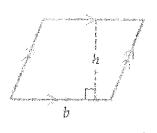
$$A = \frac{1}{2}bh$$

Trapezium



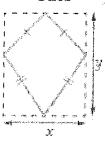
$$A = \frac{1}{2}(a+b)h$$

Parallelogram



$$A = bh$$

Kite



$$A = \frac{1}{2}xy$$

Rhombus

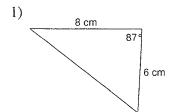


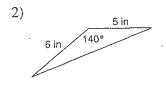
$$A = \frac{1}{2}xy$$

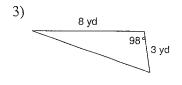
	To compare the second s		
	in the Transfer		

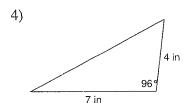
Trigonometry and Area

Find the area of each figure. Round your answer to the nearest tenth.





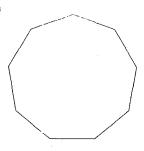




- 5) A triangle with two sides that measure 6 yd and 2 yd with an included angle of 10°.
- 6) A triangle with two sides that measure 6 m and 8 m with an included angle of 137°.

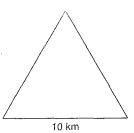
- 7) A triangle with two sides that measure 5 cm and 8 cm with an included angle of 39°.
- 8) A triangle with two sides that measure 8 ft and 7 ft with an included angle of 30°.

9)

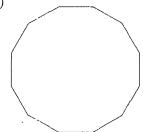


Perimeter = 108 mi

10)

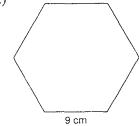


11)



Ferimeter = 144 cm

12)



13) A regular hexagon with a perimeter c 48 yd.

14) A regular pentagon 6 ft on each side.